Key

# Math III Review
## Polynomials A-APR.1

**Adding polynomials** – Add like terms, the exponents don't change!

Ex/ $(3x^2 - 4 + 2x) + (5x - 6x^2 + 7) = -3x^2 + 7x + 3$

**Subtracting polynomials** – Keep the first polynomial the same, change the subtraction to addition, and change the signs of the second polynomial. Exponents don't change.

Ex/ $(3x^2 - 4 + 2x) - (5x - 6x^2 + 7) = 9x^2 - 3x - 11$

**Multiplying Polynomials** – Each term in a polynomial has to be multiplied to each term in the other polynomial. Exponents change when terms are multiplied.

Ex/ $(2x^2 - 6x + 1)(x + 3)$

$$2x^3 + 6x^2 - 6x^2 - 18x + x + 3$$

$$2x^3 - 17x + 3$$

Ex/ $(x + 5)(x - 2)(3x + 4)$

$$x^2 - 2x + 5x - 10\ (3x + 4)$$

$$(x^2 + 3x - 10)(3x + 4)$$

$$3x^3 + 4x^2 + 9x^2 + 12x - 30x - 40$$

$$3x^3 + 13x^2 - 18x - 40$$

**Dividing Polynomials** – Can divide using synthetic or long division

**Synthetic**

$(2x^3 - 13x^2 + 26x - 24) \div (x - 4)$

$$\begin{array}{r|rrrr} 4 & 2 & -13 & 26 & -24 \\ & & 8 & -20 & 24 \\ \hline & 2 & -5 & 6 & 0 \end{array}$$

**Long Division**

$(2x^3 - 13x^2 + 26x - 24) \div (x - 4)$

$$\begin{array}{r} 2x^2 - 5x + 6 \\ x - 4\overline{)2x^3 - 13x^2 + 26x - 24} \\ \underline{(-)\,2x^3 - 8x^2} \qquad\qquad \\ -5x^2 + 26x \qquad \\ \underline{(-)\,-5x^2 + 20x} \qquad \\ 6x - 24 \\ \underline{(-)\,6x - 24} \\ 0 \end{array}$$

**Roots, Zeroes, X-Intercepts** – Are all solutions to polynomials

Finding the polynomial given the roots

Ex/ Find a 3rd degree polynomial given the roots 2 and 3$i$

$(x-2)(x-3i)(x+3i)$

$(x-2)(x^2+3xi-3xi-9i^2)$

$(x-2)(x^2+9)$

$x^3-2x^2+9x-18$

Ex/ Find the roots of $x^3-4x^2+4x-16$

-Graph the polynomial and find a zero. This polynomial crosses the x-axis at 4.

-Divide $(x-4)$ from the polynomial.

$$\begin{array}{r|rrrr} 4 & 1 & -4 & 4 & -16 \\ & & 4 & 0 & 16 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

-So the polynomial is reduced to $x^2+4$. Can use either QF or solve the square root equation.

$x^2+4=0$

$-4 \quad -4$

$x^2=-4$

$\sqrt{x^2}=\pm\sqrt{-4}$

$x=\pm 2i$

So the roots of the polynomial are 4, 2$i$, -2$i$

Review Examples

1. Which expression is equivalent to $(x+3)^3-9x(x+3)$?
   A. $x^3+27$
   B. $x^3-27$
   C. $x^3-9x^2-27x+27$
   D. $x^3-9x^2+27x+27$

$(x+3)(x+3)(x+3)-9x(x+3)$

$(x^2+6x+9)(x+3)-9x(x+3)$

$x^3+3x^2+6x^2+18x+9x+27$

$(x^3+\cancel{9x^2}+\cancel{27x}+27)-(\cancel{9x^2}+\cancel{27x})$

$x^3+27$

2. The volume of a rectangular prism is represented by the expression $(x^3 - 2x^2 - 20x - 24)$. If the length is $(x - 6)$ and the height and width are equal, what is the width of the prism?

A. $x + 2$
B. $x - 2$
C. $x + 4$
D. $x - 4$

Since $V = L \cdot W \cdot H$ divide out the length

$$\begin{array}{r|rrrr} 6 & 1 & -2 & -20 & -24 \\ & & 6 & 24 & 24 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

$x^2+4x+4$

$(x+2)(x+2)$ ← one is height the other is width

So $H \cdot W = x^2+4x+4$

3. Suppose $p(x) = x^3 - 2x^2 + 13x + k$. The remainder of the division of $p(x)$ by $(x + 1)$ is -8. What is the remainder of the division of $p(x)$ by $(x - 1)$?

A. -8
B. 8
C. 16
D. 20

$$\begin{array}{r|rrrr} -1 & 1 & -2 & 13 & K \\ & & -1 & 3 & -16 \\ \hline & 1 & -3 & 16 & -8 \end{array}$$

So $K = 8$

So now divide out $(x-1)$ from $x^3-2x^2+13x+8$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & 13 & 8 \\ & & 1 & -1 & 12 \\ \hline & 1 & -1 & 12 & 20 \end{array}$$

↑ Remainder

4. Which expression is the factored form of $x^3 + 2x^2 - 5x - 6$?

A. $(x + 1)(x + 1)(x - 6)$
B. $(x + 2)(2x - 5)(x - 6)$
C. $(x + 3)(x + 1)(x - 2)$
D. $(x - 3)(x - 1)(x + 2)$

$(x+3)(x+1)(x-2)$

$(x^2+4x+3)(x-2)$

$x^3-2x^2+4x^2-8x+3x-6$

$= x^3+2x^2-5x-6$

5. What are the zeroes of the polynomial function $y = 2x^3 - 7x^2 + 2x + 3$?

A. $\frac{1}{2}, 1, 3$ B. -1, 1, 3 C. $-\frac{1}{2}, 1, 3$ D. $-3, \frac{1}{2}, 1$

↑ graph in calc and look at where it crosses the x-axis

6. Which polynomial function has zeroes at -4, 3, and 5?

A. $f(x) = (x + 4)(x + 3)(x + 5)$
B. $g(x) = (x + 4)(x - 3)(x - 5)$
C. $h(x) = (x - 4)(x - 3)(x - 5)$
D. $k(x) = (x - 4)(x + 3)(x + 5)$

7. Which is not a factor of $x^3 - x^2 - 17x - 15$? ← graph in calc and it crosses the x-axis at -3, -1, 5 so the factors are $(x+3)(x+1)(x-5)$

A. $x - 5$ B. $x + 1$ C. $x + 3$ D. $x + 5$

8. Which of the following is not a solution of $x^4 - 3x^2 - 54 = 0$? ← graph in calc it crosses the x-axis at -3 & 3. Since those are roots divide them out of the equation

A. -3 B. 3 C. $-3i$ D. $-i\sqrt{6}$

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -3 & 0 & -54 \\ & & -3 & 9 & -18 & 54 \\ \hline & 1 & -3 & 6 & -18 & 0 \end{array}$$

$x^3-3x^2+6x-18$

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 6 & -18 \\ & & 3 & 0 & 18 \\ \hline & 1 & 0 & 6 & 0 \end{array}$$

$x^2+6=0$

Solve

$x^2=-6$

$x=\pm\sqrt{-6}$

$x=\pm i\sqrt{6}$

$(a-b)(a-b)(a-b)$

$(a^2-2ab+b^2)(a-b)$

$a^3-a^2b-2a^2b+2ab^2+ab^2-b^3$

$=a^3-3a^2b+3ab^2-b^3$

9. What is the expanded form of $(a-b)^3$?
   - A. $a^3+a^2b+ab^2+b^3$
   - B. $a^3+3a^2b+3ab^2+b^3$
   - C. $a^3-a^2b+ab^2-b^3$
   - D. $a^3-3a^2b+3ab^2-b^3$

10. The function $f$ is defined as $f(x)=6x^4+x^3+4x^2+x-2$.
    - Using the Remainder Theorem, determine if $\frac{1}{2}$ is a root of $f(x)$. Explain.

Synthetic division. If ½ is a root then the remainder when it is divided out is zero

$$\begin{array}{r|rrrrr} \frac{1}{2} & 6 & 1 & 4 & 1 & -2 \\ & & 3 & 2 & 3 & 2 \\ \hline & 6 & 4 & 6 & 4 & 0 \end{array}$$

← yes it is a root

- If $i$ is also a root, what are the other two roots?

imaginary root have a conjugate pair so another root is $-i$. The other root is a real number and when the function is graphed it shows it as $x=-2/3$

So the 4 roots are: $\frac{1}{2}, -\frac{2}{3}, i, -i$

11. For a certain polynomial function, x = 3 is a zero with multiplicity of two and x = -3 is a zero with a multiplicity of one. Write a possible equation for this function and sketch its graph.

multiplicity means two roots that are the same. So this function has two roots of 3.

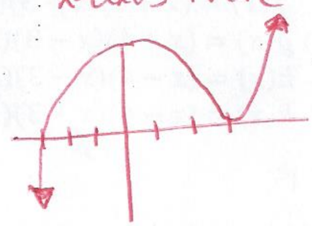
$(x-3)(x-3)(x+3)$

$(x^2-6x+9)(x+3)$

$x^3+3x^2-6x^2-18x+9x+27$

$x^3-3x^2-9x+27$

roots = 3,3,-3 so it crosses the x-axis there



12. Is $(2x-3)^3-64$ equivalent to $(2x-11)(2x+5)$? Explain your reasoning.

$(2x-3)(2x-3)(2x-3)-64$

$(4x^2-12x+9)(2x-3)-64$

$8x^3-12x^2-24x^2+36x+18x-27-64$

$=8x^3-36x^2+54x-91$

$=4x^2+10x-22x-55$

$=4x^2-12x-55$

no